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ABSTRACT

This document describes a method for deriving K factors and includes instructions for applying them to reliability prediction. Supporting rationale and background material are also included. The method was developed by the Research and Engineering Division of the Boeing Aerospace Company as an independent research and development project. Field experience data at the Line Replaceable Unit (LRU) level were the basic data used in developing the method. Other applications of this K factor approach, such as Maintainability, will be documented and released separately.

Key Words

Failure Rate

K Factor

Reliability

Prediction

Acknowledgement

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1.0 INTRODUCTION

In general, K factors (Logistic Performance Factors) are numbers which are used to adjust LRU (Line Replaceable Unit) field experience data from one environment to make predictions about the LRU performance in another environment. However this document deals only with failure rate K factors, shows that a definite need exists both for their use, and for research into their development, and details a method for calculating K factors. Sections 1 and 2 include all information necessary to understand the method and to begin to use it. Section 3 explains how the method was developed and validated, and will provide the reader with a better understanding of the usefulness and limitations of this K factor approach. Because this document deals with only one type of K factor, consider "K factor" and "**failure rate** K factor" synonymous throughout.

1.1 Objective

The initial step of this research effort was to develop and statistically validate a method whereby K factors could be calculated from field experience data. The second step was to use the method to produce a set of K factors and to further validate the method by checking these K factors against actual operational data. The third step was to see what applications in addition to failure rate prediction there would be for K factors calculated in this manner, especially in the areas of maintainability and system safety.

This report discusses the first two of these steps of the research effort. It includes step by step illustrations for applying the developed method to field experience data to produce results useful for Reliability prediction applications during the design phase of new systems. Results of the third step will be documented and released separately for each area, such as maintainability, that proves to be suitable for K factor application.

1.2 Background

Most aerospace programs are required by contract to perform complete reliability, maintainability, and system safety evaluations, and usually the contracts specify MIL-HDBK-217A dated 1965 and MIL-STD-756A dated 1963 to be used as reliability prediction guidelines. However MIL-HDBK-217A only lists failure rates and a few gross environmental K factors for some electronic piece parts, and MIL-STD-756A only lists gross environmental K factors for the group of electronic piece parts not covered in MIL-HDBK-217A (see Table 1.2-1, page 7). This means there are no K factors for use at the LRU level and only a few electronic piece part K factors. Further, considerable failure data has

been generated from current state-of-the-art equipment that contradicts the listed K factors developed from data that was collected over 15 years ago on equipment that was designed and built well before that (Reference 3).

Table 1.2-1

MIL-STD-756A Environmental K Factors

Shipboard		1.0
Manned Aircraft		6.5
Missiles		80.0
Satellite:	Launch and Boost Phase	80.0
	Orbit Phase	1.0

Current factors are too gross for prediction purposes as system configuration and environmental applications become evident early in the design stages or even before that in the project planning stage of a program. The effects of this situation are reflected in AFLCP 800-3 dated April 1973. "While failure data collection has provided historical failure rates, insufficient effort has been made to date to calculate usable K factors. As a result forecasted failure rates may be highly inaccurate with unfavorable effects extending to LCC (Life Cycle Cost) and spares computations."

Another point that was important in defining the course of this research was that more and more emphasis is being placed on using equipment similar or equivalent to a single LRU or a group of LRU's in existing aerospace systems. Typically there is abundant experience data on equipment of this type, but not necessarily in the same application or environment as the new design.

Yet another point was that existing K factors did not provide a means of estimating their own validity. In other words, a possible range of values or "confidence limits" was not given. Part of the research was devoted, therefore, to attempt to establish some sort of confidence limits.

For these reasons, in 1973 a specific effort was initiated to determine what kinds and amounts of data were available at the LRU level, and to gather this data in a form that could be used to develop K factors. Then in 1974 with the knowledge of the kinds and amounts of data available and an approximate idea of the results obtainable, the research effort concentrated on developing a valid statistical method for calculating K factors which could be used with a quantifiable amount of confidence at the LRU level.

1.3 Scope

K factors have many applications, but in this document the primary emphasis and intended use for them is in reliability prediction studies for new aerospace design applications. Limited resources have restricted this phase of the research to developing and validating a method for use at the generic system level. However, as data improves and resources become available, it may be possible to look at subfactors such as complexity, mission type, duty cycle, etc. within generic systems to improve this method (refer to Section 3.3.4, Subfactors that Impact Reliability, page 35).

The method developed and validated in this effort is based on statistical techniques taken from texts included in the reference list. The statistical techniques are straightforward and easy to use with the aid of a computer or programmable calculator, and none are new or unproven.

2.0 METHOD PRESENTATION

A K factor (K) is the ratio of the same statistic (f()) taken from data sets from two different environments (DS_a, DS_b) and represents the fractional contribution to the statistic that is solely attributable to just the environmental differences between the two data sets:

$$K = \frac{f(DS_a)}{f(DS_b)}$$

For reliability this is better illustrated as

$$K = \frac{\hat{\lambda}_a}{\hat{\lambda}_b}$$

where $\hat{\lambda}_a$ and $\hat{\lambda}_b$ are the geometric mean failure rates for data sets a and b respectively.

In this study the data sets are either failure rates or MTBF's for LRU groups and the statistic is the geometric mean of the data set.

Reliability K factors are used when a failure rate prediction is needed for a particular item, but no failure history data is available on it in the desired application. Data on the item from another application (λ_a) can be adjusted by using the appropriate K factor. If the proper K factor has already been assigned, the calculation is simply:

$$\lambda_{predicted} = K \cdot \lambda_a$$

However, if the proper K factor is not available, a sampling of failure rate data from a few LRU groups within the general equipment classification from both the new and old environments must be gathered, first level K factors calculated, and a composite K factor calculated (see figure 2.3-1, page 15). Then the failure rate prediction would again be:

$$\lambda_{predicted} = \hat{K} \cdot \lambda_a$$

In a few special cases it may be both possible and advantageous to develop just one first level K factor from data on equipment belonging to the same LRU group in question. To be possible there must be sufficient data on the specific LRU group from two environments. To be advantageous just the one failure rate prediction in that general equipment classification should be

required and there should be failure rate data on an identical LRU in the old environment. Otherwise, data on a similar LRU in the new application would be as good or better than factored data on a similar LRU in another application. In a rare case such as this the prediction equations would be,

$$K = \frac{\lambda_{New}}{\lambda_{Old}}$$

$$\lambda_{predicted} = K \cdot \lambda_a$$

2.1 General Technique

The geometric mean or nth root of the product of n values is the basic technique upon which this K factor method is based. Other measures of central tendancy were tried and are discussed in Section 3.1, (page 22) along with justification for choosing the geometric mean.

Calculating the geometric mean is most conveniently done by summing the logarithms of all the data points, dividing the sum by the number of data points and taking the antilog of the quotient to give the geometric mean. Further calculations, which are outlined in Section 2.3 (page 11), give confidence limits to the mean and subsequent K factors. Appendix I is a calculator program which can be used to do all of the above mentioned calculations. Outputs are geometric mean and mean confidence limits for any input data set. By using a computer or programmable calculator, time can be saved and chance for error in the many calculations is greatly reduced.

It is important to note that by using the geometric mean approach, failure rates or MTBF's work equally well as inputs, that is the resulting means and limits are exact reciprocals, a result that is not possible by any other averaging technique.

2.2 Assumptions

Field experience data is believed to be the best available failure data source. Nonetheless, it has some known drawbacks, for example, there are errors in reporting, individual times-to-failure are not known, and the distribution of failure rates is uncertain. Furthermore, field experience data does not reflect "true" or "absolute" reliability but, rather, reliability as it is affected by other factors. A modified Bayesian approach to the problem was therefore adopted in which, a priori, certain assumptions concerning the data were made with the reservation that subsequent research may require modification of, or may even invalidate, the assumptions. One assumption discussed in

the previous section was that the geometric mean was the best measure of central tendency. In addition certain other basic assumptions have been made concerning the data used and its applicability to K factor determination.

(1) Experience data sets reflect an integration of all subfactors which affect reliability statistics.

(2) Reliability statistics vary primarily due to environmental effects, while other contributing effects tend to cancel when K factor ratios are taken.

(3) LRU's in a general equipment class are all affected similarly by changes in application, such that a single composite K factor will adequately represent the entire class.

(4) A direct relationship exists between failures and operating hours (constant failure rate).

(5) Failure rates are lognormally distributed.

It is important to remember that most analysis and trade studies which use these factors are made for comparative purposes early in the program, rather than for absolute values. Therefore certain errors in these K factors will not obscure the trade study results where the error in other considerations is often larger. However these assumptions do bring in some error, and for this reason they are discussed further in Section 3.3, Problem Areas.

2.3 Method Detailed

The following equations specify how the geometric mean (G. M.) and G. M. confidence intervals of a data set are calculated.

(1) Geometric mean, $\hat{a}$:

$$G.M. = \hat{a} = \sqrt[n]{\prod_{i=1}^{n} a_i} = 10^{\left(\frac{\sum_{i=1}^{n} \log_{10} a_i}{n}\right)} \quad \text{Note: } a_i = \lambda_i, MTBF_i, \text{or } K_i$$

(2) Log Variance, s_L^2:

$$s_L^2 = \frac{\sum_{i=1}^{n}\left(\log_{10} a_i - \log_{10}\hat{a}\right)^2}{n-1} = \frac{n\sum_{i=1}^{n}\left(\log_{10} a_i\right)^2 - \left(\sum_{i=1}^{n}\log_{10} a_i\right)^2}{n(n-1)}$$

(3) Upper (a_u) and Lower (a_L) G.M. confidence limits at $1-\alpha$ level of confidence, such that

$$P\left[\hat{a}_L < a = 10^{(\log_{10}\hat{a})} < \hat{a}_u\right] = 1-\alpha$$

$$\hat{a}_u = 10^{(\log_{10}\hat{a}+\varepsilon)}$$ where, $\varepsilon = T\sqrt{s_L^2/n}$

$$\hat{a}_L = 10^{(\log_{10}\hat{a}-\varepsilon)}$$ and, $T = t_{1-\alpha,\ n-1}$*

or, $T = t\alpha/2,\ n-1$

(4) K factor, K; and Upper/Lower confidence limit K factors, K_u/K_L:

$$K = \frac{\hat{a}}{\hat{b}}, \qquad K_u = \frac{\hat{a}_u}{\hat{b}_L}, \qquad K_L = \frac{\hat{a}_L}{\hat{b}_u}$$

(5) Composite K factors, $\hat{K}$; and their Upper/Lower confidence limit K factors, $K_u/\hat{K}_L$:

$\hat{K}$ = G.M. of $K_1, K_2 \ldots K_n$ (see equation #1)

$\hat{K}_u = \hat{a}_u$ and $\hat{K}_L = \hat{a}_L$ based on $K_1, K_2, K_3 \ldots K_n$

*Most t tables list α versus ν, but Appendix II lists $1-\alpha$ versus $\nu = n-1$, where ν is degrees of freedom.

The foregoing equations show the relationships between raw data and their resulting K factors. Initially all failure rate data are sorted by LRU groups, each of which is defined by its unique construction and application. Generally the requirements are such that all data associated with a particular LRU group must come from LRU's which are at least similar if not identical in construction and application/environment. Then data from each LRU group is processed using equations 1, 2, and 3, yielding geometric mean and confidence limits for each group. (See Figure 2.4-2, page 21)

Next, first level K factors are developed, first by matching pairs of LRU groups that are nearly identical in construction but different in environment and secondly by applying equation #4 to the mean and limits previously developed for each matched pair. It is assumed that the LRU's in the matched LRU groups would have a common G.M. failure rate if used in the same environment, therefore the ratios (or K factors) developed using equation #4 measure the relative increase (decrease) in failure rate due to a

more severe (less severe) environment. The resulting first level K factors can be identified by LRU group and two associated environments. (It is important to note which environment is used as the base when equation #4 is applied.) (See Table 2.3-1, page 14)

Often when only one failure rate prediction is needed, this is as far as the process needs to be followed (See Section 2.4 Example #2, page 19 for an example using first level K factors.) But for the majority of cases, a more general type K factor, described in this document as a composite K factor, would be more useful in mass application on a large program. These composite K factors are developed from the first level K factors by grouping them by identical environment combinations and then by further sub-dividing these groups into subgroups which are defined by the general equipment classification of the LRU groups. The order in which these first level K factor groups are sorted is not important as long as the members of each resulting subgroup have common classes of hardware and identical environment combinations. Equations 1, 2, and 3 are then applied to the first level K factor values to arrive at composite K factors and their confidence limits. (See Tables 2.3-1, page 14 and 2.4-1, page 18). This method of grouping permits equipment class composite K factors to be developed from first level K factors of LRU groups that do not have common G.M. failure rates. This is possible because each first level K factor is a ratio or index of severity which is independent of the gross magnitude of the failure rates. Therefore this process enables reliable K factors to be developed from a minimum sampling of failure rate data, as illustrated in Figure 2.3-1, page 15.

The upper (K_u, $\hat{K}_u$) and lower (K_L, $\hat{K}_L$) confidence interval limits (K factors) are developed to give the user an idea of the dispersion of the failure rates used to calculate the K factors. A "worst case" condition was used for calculating confidence limits in which it was assumed that the two data sets would have actual values at the opposite extremes. If a 90% confidence level is chosen to calculate these K factors (typical for this type of calculation), this means that there is a .9 probability that the true K factor lies between the upper and lower confidence limits K factors. However, by the strict mathematical definition, it does not mean that there is a .9 probability that the actual failure rate of an LRU in a new application will be within these limits, although results of empirical testing do indicate that more than 90% of actual values will be within these limits when failure rate data on the same LRU is factored.

Table 2.3-1
Aircraft Electronics
Composite K Factor Work Table

LRU		Mil. Transport F.R.	Mil. Transport K	Civil Transport F.R.	Civil Transport K	Fighter F.R.	Fighter K	Bomber F.R.	Bomber K
UHF/VHF	L.L.	5.43		.58	0.05	$\boxed{12.50}$ *	1.04	8.00	0.67
	G.M.	8.07	1	1.11	0.14	$\boxed{17.10}$	2.12	10.40	1.29
	U.L.	12.00		2.26	0.42	$\boxed{23.30}$	4.29	13.60	2.50
Radar Altimeter	L.L.	3.01		0.40	0.04	11.70	1.28	2.72	0.30
	G.M.	5.25	1	0.59	0.11	18.10	3.45	5.16	0.98
	U.L.	9.16		0.86	0.29	28.00	9.30	9.76	3.24
IFF	L.L.	2.63				4.46	0.73	1.40	0.23
	G.M.	4.02	1			6.41	1.59	3.94	0.98
	U.L.	6.14				9.21	3.50	11.10	4.22
TACAN	L.L.	8.25				11.80	0.91	7.09	0.55
	G.M.	10.30	1			16.30	1.58	12.80	1.24
	U.L.	12.90				22.60	2.74	23.10	2.80
Composite Electronics	L.L.				0.06		1.34		0.94
	G.M.		1		0.12	**	$\boxed{2.07}$		1.11
	U.L.				0.27		3.19		1.33

* From Figure 2.4-2, page 21 and Table 3.1-1, page 24

** See Table 2.4-1, page 18

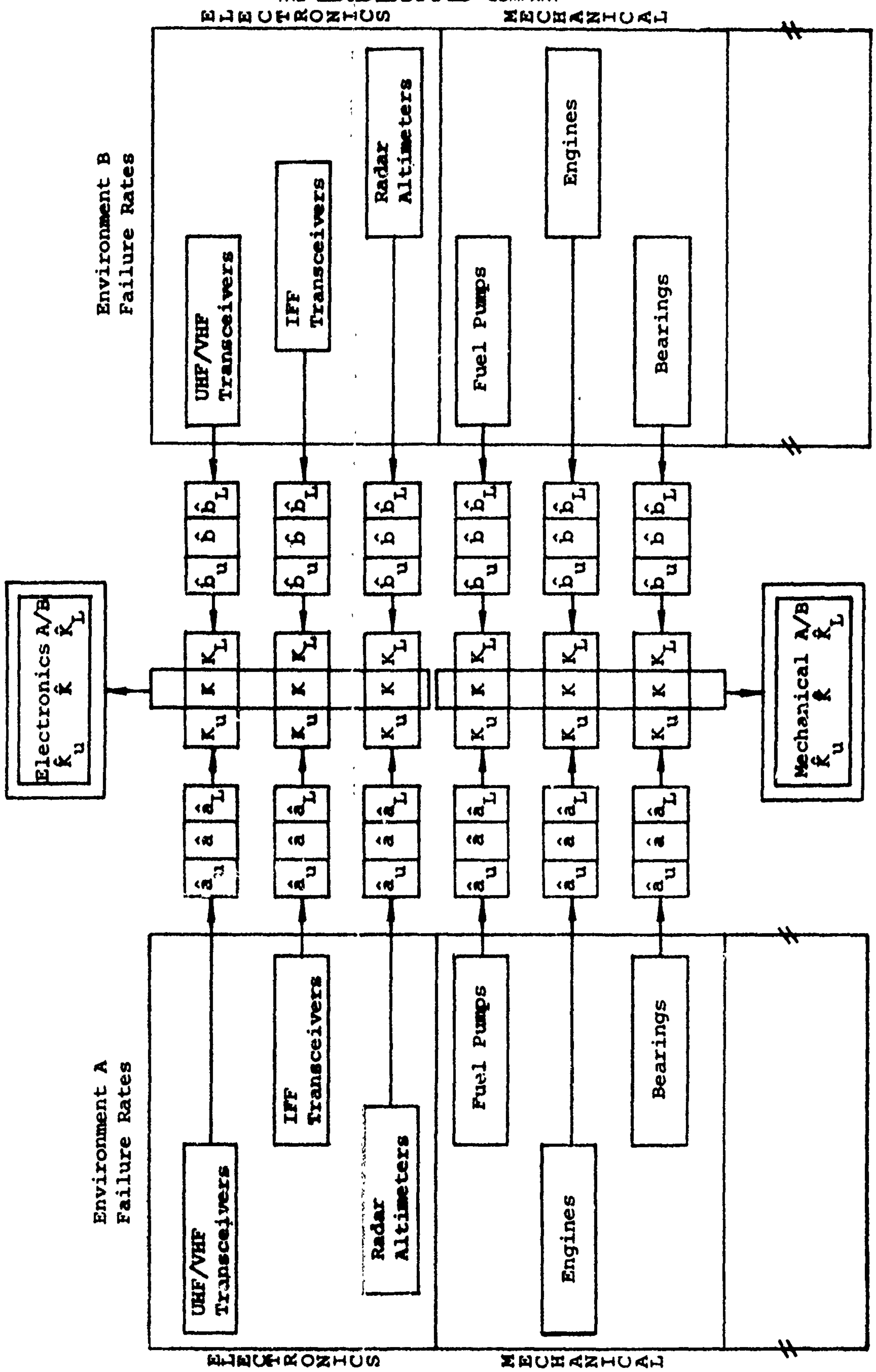

Figure: 2.3-1
Failure Rate Data Grouping for Composite K Factor Development

Throughout this report, common logarithms and corresponding powers of 10 have been used because of the ease in accessing tabular values, however natural logarithms and the exponential function work equally as well. Intermediate results, specifically the mean logarithms and standard deviations, are not the same, but the end results are identical, therefore it is important that one or the other approach be used exclusively. In fact there are many ways that the calculations, defined by the K factor equations at the beginning of this section, can be made, and the best way will depend on the user's individual situation. For convenience, a calculator program that performs geometric mean, confidence limit and frequency boundary limit calculations is included as Appendix I, and a simplified manual process is detailed in example #2 of Section 2.4 (page 20). If several K factors are to be calculated, computer aided processing will reduce the time required and will greatly reduce the chance for arithmetic errors.

2.4 Application to Reliability

The prime objective of the research effort was to develop and validate a useful K factor development method. This section is devoted to applying the developed method to reliability prediction.

A flow diagram, Figure 2.4-1 (page 17), illustrates how this method would be used in reliability prediction. Referring to the diagram, as soon as the need for a failure rate or MTBF prediction has been established, it must be determined what is the best kind of reliability data available. If the best data is failure history data on the same/similar item in another environment, a K factor adjustment by the method described herein would produce the desired results. In most cases a composite K factor table similar to Table 2.4-1 (page 17) would contain the appropriate K factor. However some programs have specific definitions of failure that are not compatible with the general form. In such a case a whole new set of composite K factors would need to be calculated by processing data according to the definition of failure set by the program. These program composite K factors may all be researched and calculated at one time to reduce the number of manhours needed to complete an entire set of K factors, and then logged in a reference file (the EAC maintains such a file) for use on the program and possibly for future programs. Sometimes only one K factor is needed and in these cases only one data set from each of the new and old applications need to be gathered to produce the K factor. At any rate, the end result of using the procedure illustrated in Figure 2.4-1 (page 17) is always a reliability prediction with the highest possible confidence.

RELIABILITY PREDICTION FROM
FIELD EXPERIENCE DATA

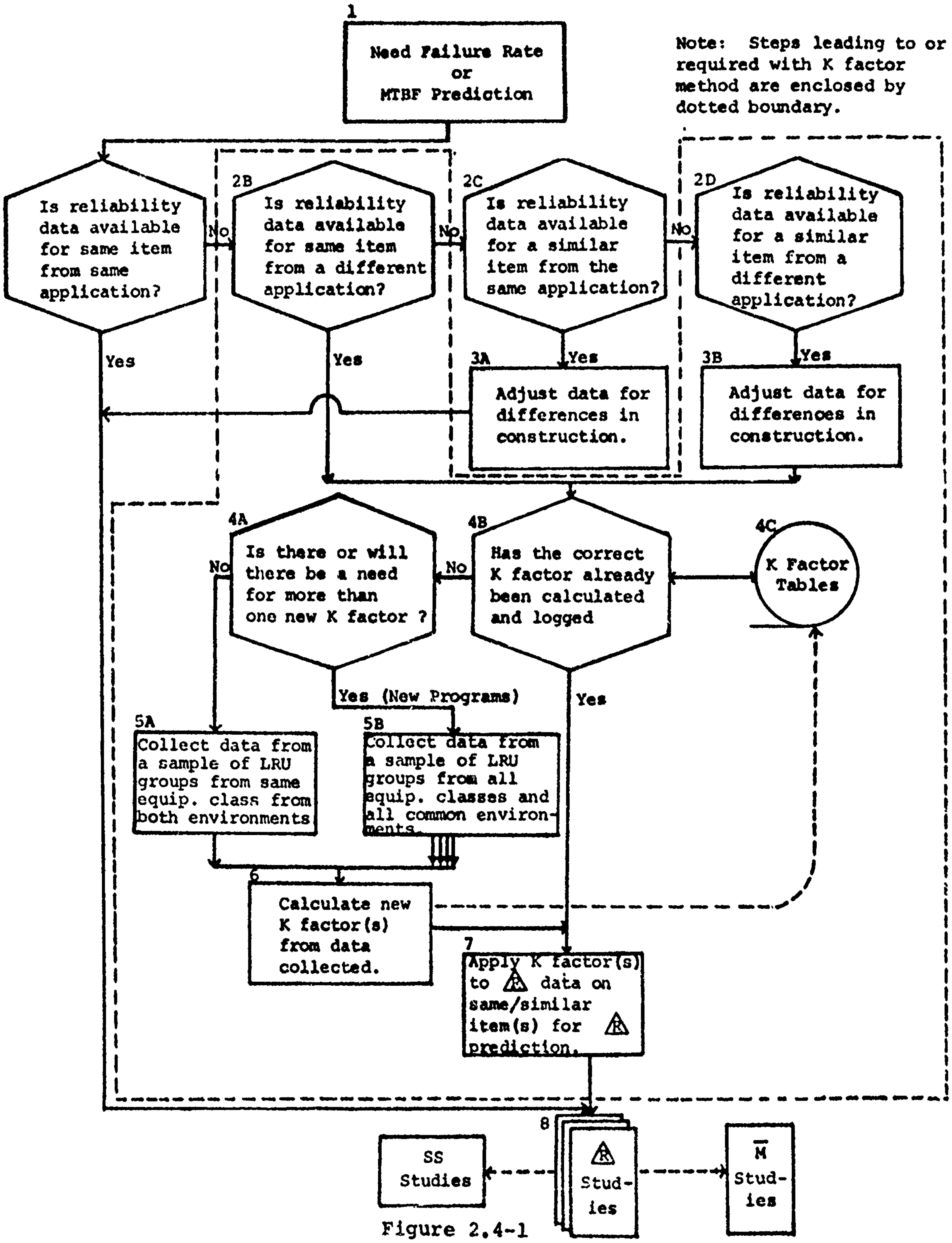

Figure 2.4-1

Example 1

Given: Need a failure rate for an AN/ARC-109 UHF transceiver for fighter aircraft application.

Assume that the transceiver has never flown in a fighter, but that 4.838 failures per 1000 flight hours were reported against the same transceiver in a C-5A military transport application.

Because a transceiver is electronic equipment, the electronics composite K factor for military transport to fighter from Table 2.4-1 (below) will be used and the calculations are as follows:

$$\lambda_{predicted} = \lambda_{C-5A} \times \hat{K}_{f/m.t.} = 4.838 \times 2.1 = 10.16 \text{ failures per 1000 flight hours}$$

Table 2.4-1 is a preliminary composite K factor table formulated from actual data by the method developed in this study and used in this reliability prediction example. Note that K factors for military transport are unity and that all other application K factors are shown relative to the military transport. When such a table is developed for a reliability study on a new program, the new application could be used as the base, or a complete cross-reference table could be set up for each equipment class with no common base necessary.

Table 2.4-1

Aircraft Composite K Factors

Application	Mechanical/ Hydraulic			Electro-Mechanical			Electronic		
	$\hat{K}_u$	$\hat{K}$	$\hat{K}_L$	$\hat{K}_u$	$\hat{K}$	$\hat{K}_L$	$\hat{K}_u$	$\hat{K}$	$\hat{K}_L$
Military Transport		1			1			1	
Bomber	8.1	3.2	1.1	2.0	1.0	0.5	2.7	1.5	1.1
Fighter	4.6	2.0	0.8	3.0	1.5	0.7	3.2	$\boxed{2.1}$*	1.3
Helicopter	1.8	0.8	0.3	5.9	2.9	1.4	1.4	0.9	0.5
Commercial Transport	1.8	0.6	0.2	1.6	0.4	0.1	0.2	0.1	0.1

* From Table 2.3-1

In reality, the AN/ARC-109 had been used in the F-111A fighter aircraft with a field demonstrated failure rate of 12.68 failures per 1000 flight hours, well within the expected range of the predicted value of 10.16 failures per 1000 flight hours.

The Experience Analysis Center (EAC) of the Boeing Aerospace Company, maintains a log and file of all K factors developed using this approach. It may already have single K factors or K factor tables for different programs. The EAC should be contacted before indiscriminately using any K factor, since the definition of failure may vary from program to program and for different data sets.

Example 2

Given: Need a failure rate for an AN/ARC-109 UHF transceiver for fighter aircraft application.

The basic requirements are the same here as in Example 1, but with one added restriction - assume that the military transport to fighter composite K factors have not been calculated, therefore Table 2.4-1 (page 17) cannot be used. Now a UHF transceiver is electronic equipment and according to assumption #3 Section 2.2 (page 11) a composite electronics K factor would be applicable, as illustrated in Example 1, but assume there is sufficient data on both transport and fighter UHF/VHF transceivers to produce an accurate first level transceiver K factor, as is really the case here. Then first because the item in question belongs to the same LRU group as other transceivers, and secondly because it takes much more data to calculate a composite K factor, a first level transceiver LRU group K factor would be best, in this case. (On a large program where a full set of composite K factors would be available, the electronics K factor would be used, as in Example 1, to eliminate retrieving additional data.)

The next step is to collect and process transceiver failure rate data from both applications. Figure 2.4-2 (page 21) lists such data and shows the necessary calculations for processing the fighter data. The same steps were used to calculate mean and mean confidence intervals for the military transport transceivers, but only the results are shown. The prediction of 10.22 is again well within range of the actual rate for this transceiver on the F-111A fighter aircraft of 12.68.

The following steps were used in Figure 2.4-2 (page 21) to calculate the data set means and mean confidence limits. The steps are marked with numbered circles in the figure for ease in following the procedure.

1. List failure rates.
2. List the logarithms of the failure rates.
3. Sum the logarithms.
4. Divide the sum by the number of entries.
5. Take the antilog of the quotient to get the geometric mean.
6. List the differences between the logs of the individual failure rates and the log of the geometric mean.
7. Square and list the difference for #6.
8. Sum the squares.
9. Divide the sum by one less than the number of entries to obtain the variance.
10. From the t-table (Appendix II) find the value of t corresponding to the desired confidence level, 90%, and the appropriate degrees of freedom, n-1.
11. Compute ε, the deviation from the sample mean, from the formula $\varepsilon = t\sqrt{s_L^2/N}$.
12. Compute the upper 90% mean confidence limit.
13. Compute the lower 90% mean confidence limit. (Repeat steps 1 through 13 for second data set.)
14. Compute K factors, K, K_u, K_L.
15. Compute predicted failure rate.
16. Log K factors.

i	A/C	$\lambda_i \times 10^3$ (1)	$\log \lambda_i$ (2)	$\log \lambda_i - \log \hat{\lambda}$ (6)	$(\log \lambda_i - \log \hat{\lambda})^2$ (7)
1	F-4B	25.78	1.411283	.179519	.032227
2	F-4E	15.66	1.194792	-.036971	.001367
3	F-111A	12.68	1.103119	-.128644	.016549
4	F-111F	6.27	0.797268	-.434495	.188786
5	A-7A	22.24	1.347135	.115371	.013310
6	A-7B	23.77	1.376029	.144265	.020812
7	A-7D	18.67	1.271144	.039380	.001551
8	A-7E	22.56	1.353339	.121575	.014780
Sums		(3)	9.854109	(8)	.289382

(4) $\log \hat{\lambda} = \frac{\Sigma \log \lambda_i}{n} = \frac{9.854109}{8} = 1.231764$

(5) $\hat{\lambda} = 10^{(\log \hat{\lambda})} = 10^{(1.231764)} = 17.05$

(9) $s_L^2 = \frac{\Sigma(\log \lambda_i - \log\hat{\lambda})^2}{n-1} = \frac{.289382}{7} = 0.041340$

(10) At 90% confidence level, $t_{90\%,7} = 1.895$ (see Appendix II)

(11) $\varepsilon = T\sqrt{s_L^2/n} = 1.895\sqrt{.041340/8} = 0.136223$

(12) $\hat{\lambda}_u = 10^{(\log \hat{\lambda} + \varepsilon)} = 10^{(1.231764 + .136223)} = 23.33$

(13) $\hat{\lambda}_L = 10^{(\log \hat{\lambda} - \varepsilon)} = 10^{(1.231764 - .136223)} = 12.46$

UHF/VHF Transceivers Military Transport $\hat{\lambda} = 8.07,\ \hat{\lambda}_u = 12.0,\ \hat{\lambda}_L = 5.43$

UHF/VHF Transceivers Fighter $\hat{\lambda} = 17.05,\ \hat{\lambda}_u = 23.33,\ \hat{\lambda}_L = 12.46$

AN/ARC-109 C-5A Failure rate 4.838

(14) $K = \frac{\hat{\lambda}_f}{\hat{\lambda}_{m.t.}} = \frac{17.05}{8.07} = \underline{2.11},\ K_u = \frac{\hat{\lambda}_{uf}}{\hat{\lambda}_{Lm.t.}} = \frac{23.33}{5.43} = 4.30,$

$K_L = \frac{\hat{\lambda}_{Lf}}{\hat{\lambda}_{um.t.}} = \frac{12.46}{12.0} = 1.04$

(15) $\lambda_{predicted} = \lambda_{C-5A} \times K = 4.838 \times 2.11 = \underline{10.22}$

Figure 2.4-2
Fighter Aircraft Transceivers

3.0 VALIDATION

This section includes all of the mathematical approaches investigated for possible use in K factor development, and the acceptance tests used to test the different approaches, plus discussion of problems encountered in this K factor research effort.

3.1 Methods Investigated

The following techniques for processing field experience data to calculate K factors were tested and evaluated, and will be discussed individually in this section.

- Arithmetic Mean
- Linear Correlation
- Non-Linear Correlation
- Forced Correlation
- Geometric Mean

3.1.1 Arithmetic Mean

In the 1973 phase of this research effort, the arithmetic mean of each of the data sets accumulated was caculated to get a quick estimate of the K factors that could be produced. The arithmetic mean is symbolized as follows:

$$\bar{a} = \frac{\sum_{i=1}^{n} a_i}{n}$$

This measure of central tendancy is the simplest, but is weighted to a great extent toward the high end and produces results derived from failure rates that are not equivalent to results derived from MTBF's (see Figure 3.2-2, page 30, and Table 3.1-1, page 24).

Because of the bias and the resulting non-equivalence of results, this method was rejected.

3.1.2 Linear Correlation

The linear correlation technique gives the slope of a straight line approximation to the data points, plus the arithmetic mean is an intermediate result, but the technique is more complicated than simply an arithmetic mean. Also a correlation factor can be calculated to give a more quantitative judgement as to how the points fit the straight line approximation. However, because the slope of the line is not useful in any practical application and the mean has the same bias mentioned in Section 3.1.1, (above), this method was also rejected.

3.1.3 Non-Linear Correlation

Two curve types, exponential $Y_c = ab^x$ and power curve $Y_c = ax^b$, were tested by the least squares method to try to approximate the data points. The power curve appeared to fit the data better than the exponential and also better than the linear correlation. Also the geometric mean was an intermediate result in the calculations and appeared to be more centrally located than the arithmetic mean. However, no explanation (except for random chance) could be made for the powers of x that were calculated, and because this method is quite complex it was also rejected.

3.1.4 Forced Correlation

The same power curve approximation described in Section 3.1.3 (above) was again tried, but this time the parameter b was set at 1 in all cases, such that the result is forced to a constant rate of the form $Y_c = ax$, where $a = \lambda$ when the statistics are failure rates. This method has the same complexity as the non-linear correlation, and again there is no significant practical advantage.

It should be noted that the linear, non-linear, and forced correlations were all curvilinear attempts at representing sets of data pairs. Because of the nature of the data and the assumptions that were made, namely assuming a constant failure rate, the data is really only one dimensional. Therefore these techniques yielded some results that were either invalid or had no application, and they were paid for by added complexity.

3.1.5 Geometric Mean

The geometric mean approach selected in this effort is actually the nth root of n products approach, calculated using logarithms as described in Sections 2.3 (page 11) and 2.4 (page 16). The geometric mean has the advantage of being less biased toward the high end than the arithmetic mean. This is true because the geometric mean is the mean, median, and mode of the logarithms of a perfect lognormal distribution (see Figure 3.2-2, page 30). The reliability data investigated appears to be distributed lognormally. An example of a chi-square test indicating that the data is distributed lognormally is included in Section 3.2.3 (page 27). It is felt that failure rates and MTBF's are distributed lognormally because they are actually ratios of failures versus time, bounded by zero on the low end and unbounded above which forces them to be skewed to the right. Also, all of the numbers from which K factors will be developed will be ratios such as failure rates (failures per 10^n hours), MTBF's (hours per failure), maintenance actions per failure, maintenance manhours per maintenance action, etc., and according to the statistical texts listed as references 6, 7, and 8, the geometric

mean is especially useful when applied to pure ratios such as these. This is true because it makes no difference which way the ratio is taken, the results are equivalent. For example, take the illustration of the geometric mean calculation for the eight fighter UHF/VHF transceivers from Example 2 in Section 2.4 (page 21). Table 3.1-1 summarizes those results, plus results for the corresponding MTBF based calculations for the geometric mean and arithmetic mean.

Table 3.1-1
Reciprocals of Geometric Mean vs Arithmetic Mean

DATA POINTS

	λ	MTBF=1/λ x 1000
1	25.78	38.79
2	15.66	63.86
3	12.68	78.86
4	6.27	159.49
5	22.24	44.96
6	23.77	42.07
7	18.67	53.56
8	22.56	44.32

Note: All λ's are in failures per 1000 hours and all MTBF's are in hours per failure.

	1	2	3	4	5	6	7
	Geometric Mean Approach			Arithmetic Mean Approach			
	λ	MTBF	1/MTBF=λ'	λ	MTBF	1/MTBF=λ'	1/λ=MTBF'
Mean	17.055	58.632	17.055	18.456	65.714	15.218	54.182
Upper 90% Limit	23.334	42.855	23.334	22.861	38.842	25.746	43.743
Lower 90% Limit	12.466	80.217	12.466	14.051	92.588	10.800	71.169

By using the G.M. approach the failure rate and MTBF results are all equivalent (Column 1 equals Column 3). However, under the Arithmetic Mean Approach, Column 4 does not equal Column 6 nor does Column 5 equal Column 7 and this will always be the case. Here they differ by as much as 40%, but many examples have been found where the difference is more than 100%. Therefore, if the arithmetic mean approach were used, two different sets of reliability K factors would have to be developed, one for use with failure rates and one for MTBF's. Likewise, similar situations would result for other RM&SS K factors, all of which are based on ratios that could just as easily be interchanged.

The geometric mean eliminates this problem by producing results that are compatible no matter how the ratio is taken. The geometric mean is the best measure of central tendency for a log-normal distribution. Further, the geometric mean is straightforward and lends itself easily to calcuation of mean confidence limits and expected frequency distributions of the data points, both of which are necessary to make objective judgements concerning the data collected and the K factors produced. For these reasons the geometric mean has been selected as the averaging technique for K factor development.

3.2 Acceptance Tests

The initial effort in 1974 was directed toward trying to validate a K factor technique, and the initial hypothesis tested was as follows:

> H_O = This group of observed failure rates is a sample from a population having a failure rate which is approximately the mean of the observed failure rates.

Acceptance would constitute validation of the technique, but it must be recognized that the classical dilemma existed, mainly the double risk of accepting a false hypothesis (β) or rejecting a true one (α).

A search of various statistical texts was made to identify methods of testing hypothesis. In addition, the problem was discussed with various people knowledgeable in the fields of statistical methods and reliability. Several possible testing methods were examined of which three at first appeared promising. These were explored in more detail, and are outlined in the following sections.

3.2.1 Cumulative Frequency Distribution, "d" Test.

A cumulative-percentage histogram is drawn for the observations in a sample of failure rates. Then two parallel polygons are drawn above and below the histogram at a distance which depends

upon the level of confidence desired to support the statement that "the cumulative frequency distribution of the population is in this band." Figure 3.2-1 (**below**) shows a basic cumulative-percentage histogram for failure rates of UHF/VHF transceivers with 95% confidence limits.

This test has the advantage of establishing confidence limits, but otherwise it is not particularly attractive. For instance, it does not directly test the stated hypothesis; and it presents the data in a form, cumulative frequency polygons, which is not generally used in this sort of application and would be unfamiliar to users.

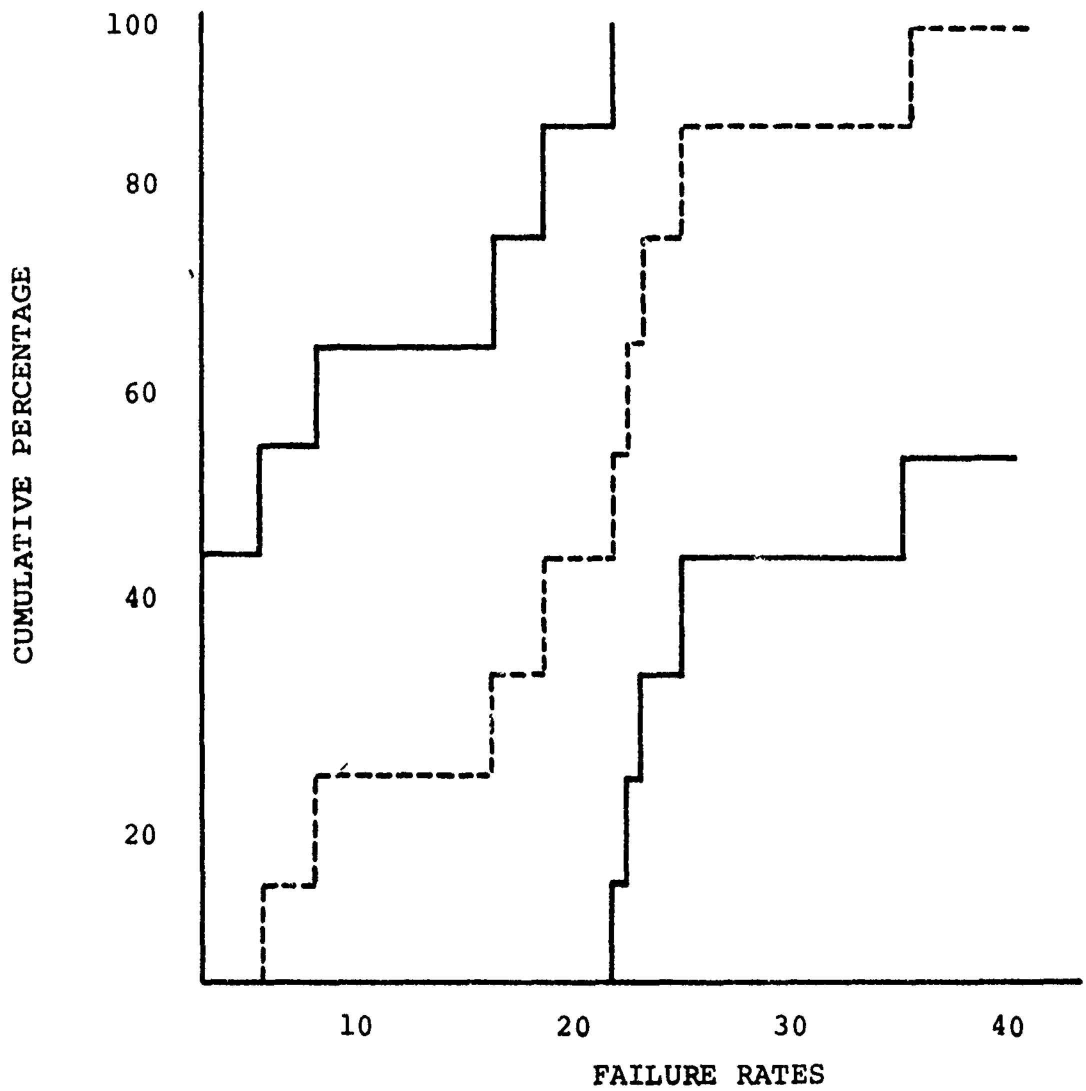

Figure 3.2-1
UHF/VHF Transceiver
Cumulative Frequency Histogram

3.2.2 Significant Ratio, "t" Ratio

This test provides an estimate of the probability, at a predetermined level of confidence, that a sample could have come from a certain population. The t ratio is:

$$t = \frac{\bar{x} - \mu}{\hat{\sigma}_{\bar{x}}}$$

where $\bar{x}$ is the sample mean, μ is the population mean, and $\hat{\sigma}_{\bar{x}}$ is an estimate of the standard error based on an estimate of the population standard deviation.

For the data at hand neither the population mean, μ, nor the individual values which make up $\bar{x}$ are known. Although estimates of these values can be drived, the net result is estimates of estimates, leading to such a degree of uncertainty that the usefulness of this test was doubtful. Also the test is most appropriate when applied to a normal distribution, whereas the distribution of the data at hand is not a normal distribution. No illustration of this test is given.

3.2.3 Chi-square (χ^2) test,

The chi-square test is useful in a variety of cases. It can be used to compare an observed parameter of a sample with the corresponding known or estimated parameter of the population from which the sample was taken.

$$\chi^2 = \frac{\sum_{i=1}^{n} (\bar{x} - \mu)^2}{\mu}$$

The value of χ^2 thus obtained is compared with the expected value of χ^2 determined by the sample size and desired confidence level taken from a chi-square table of values. This will give a probability that the sample came from the population it was assumed to be from. For the example shown in Table 3.2-1 (page 28), $\bar{X}$ would be the failure rate of an LRU in one type/model aircraft, and μ would be the sample mean failure rate for all aircraft in the sample.

A typical calculation for the chi-square test is shown in Table 3.2-1 (page 28). Referring to the table, the probability of the sample coming from the assumed population is .001, hence the hypothesis would be rejected. However an examination of a plot of the failures as a function of operating hours indicates that rejection may be the wrong conclusion.

I.D. No.	Application A	Operating Hours	Failures	MTBF	Failures/ 10^3 O.H. $\overline{X}$	$\overline{X} - \mu$	$(\overline{X} - \mu)^2$	$\frac{(\overline{X} - \mu)^2}{\mu}$
F-5	F-4B	106,276	2740	39	25.78	4.31	18.57	.86
F-6	F-4E	100,319	1571	64	15.66	-5.81	33.76	1.57
F-8	F-111A	13,950	177	79	12.68	-8.79	77.26	3.60
F-9	F-111F	6,055	38	159	6.27	-15.20	231.04	10.76
F-1	A-7A	63,905	1421	45	22.24	.77	.59	.03
F-2	A-7B	99,646	2369	42	23.77	2.30	5.29	.25
F-3	A-7D	38,508	719	54	18.67	-2.80	7.84	.37
F-4	A-7E	156,763	3537	44	22.56	1.09	1.19	.06
	TOTAL	585,422	12572					
CLASS MTBF = 47 CLASS FAILURE RATE = 21.47 = μ							X^2 = P =	17.50 .001

TABLE 3.2-1
UFH/VHF - FIGHTERS
CHI-SQUARE TEST

While only one example has been shown, it is typical of the results obtained by the chi-square test. The test was applied 19 times with the result that the hypothesis was rejected ($P < 0.01$) ten times, accepted marginally ($0.01 < P < 0.05$) four times and accepted ($P > .05$) only five times.

Hypothesis testing previously discussed, tacitly assumed the data were approximately normally distributed, but frequency plots of the data show that normality may be a poor assumption, (see Figure 3.2-2, page 30). Upon further examination of the data, the lognormal distribution appeared to be the best candidate for further testing. Another application of the chi-square test, testing the expected distribution of data points instead of the expected values, produces acceptable results. The hypothesis for this test is: H_o = The data are lognormally distributed, and the statistic tested is:

$$\chi^2 = \Sigma \frac{(f-f_e)^2}{f_e}$$

where f is the actual partition frequency, and f_e is the expected partition frequency.

By using the common logarithms of the data points (already listed for calculating the geometric mean) and a standard normal probability table, Appendix III, it is easy to calculate the partition boundary limits, the actual partition frequencies, and the expected partition frequencies.

The partition boundary limits are defined by the following equation:

$$\text{partition boundary limit} = 10^{(\log_{10}(\hat{x}) \pm z s_L^2)}$$

where z is a function of α taken from a standard normal table, $\hat{x}$ is the geometric mean, and s_L^2 is the log variance defined in Section 2.3 (page 11). The partition boundary limits are also outputs of the computer program for calculating the geometric mean, Appendix I.

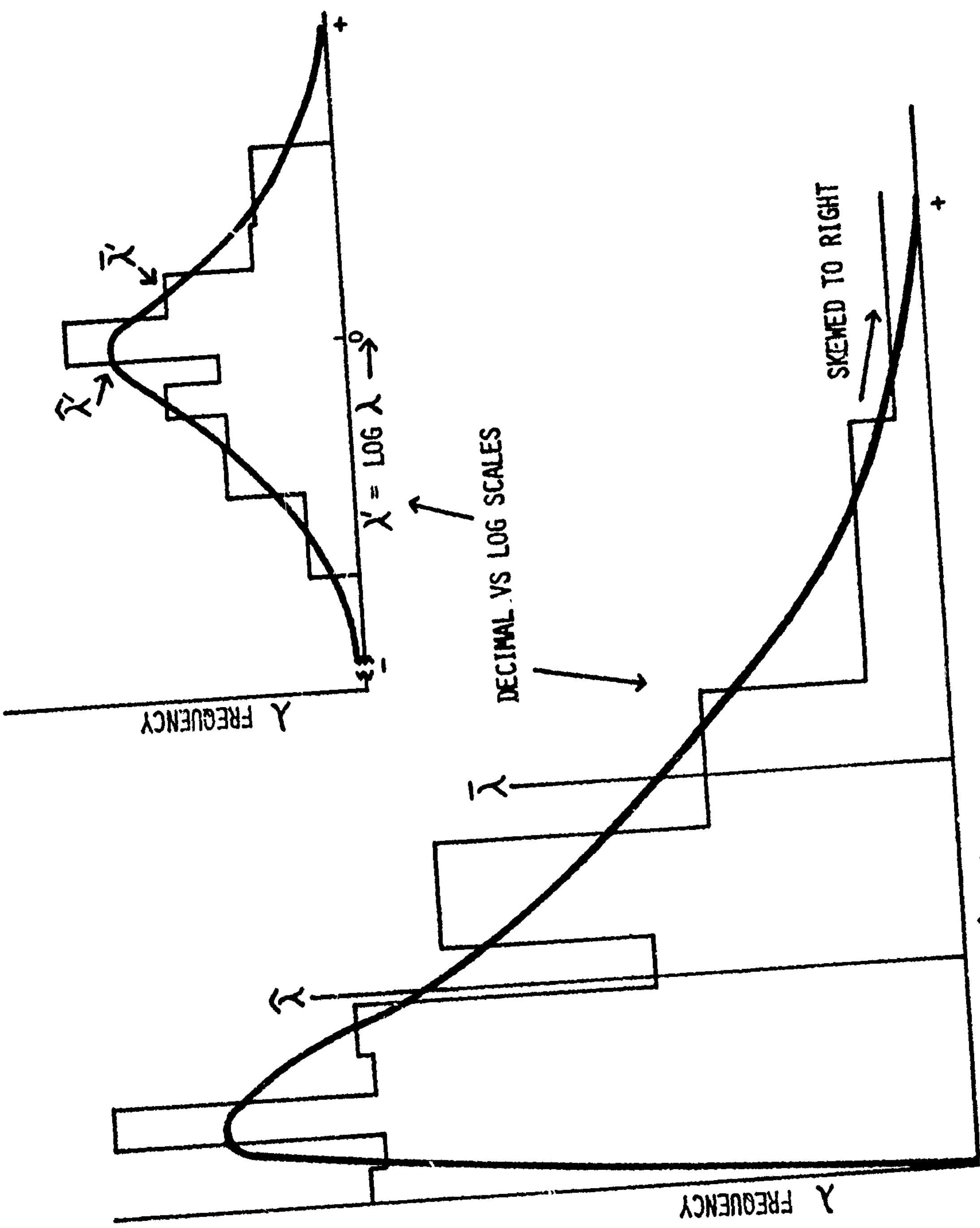

Figure 3.2-2
Frequency Distribution for 87 Gyroscope Failure Rates

Table 3.2-2

Chi-square test on frequency distribution of 87 gyroscope failure rates.

n	z	Partition Lower Limit	f_e	f	$\frac{(f-f_e)^2}{f_e}$
1	1.645	5.718	4.35	4	0.028
2	1.0365	2.832	8.7	10	0.194
3	.6745	1.864	8.7	6	0.838
4	.3854	1.335	8.7	10	0.194
5	.1256	.989	8.7	14	3.229
6	-.1256	.740	8.7	6	0.838
7	-.3854	.548	8.7	9	0.010
8	-.6745	.393	8.7	7	0.332
9	-1.0365	.258	8.7	9	0.010
10	-1.645	.128	8.7	6	0.838
11	$-\infty$	0	4.35	6	0.626
				87	$\chi^2 = 7.137$ P > .50

$\hat{x} = .8555$ $\log_{10}\hat{x} = .06727$ $s_L^2 = .5015$ $v = n-3 = 8$

The results of the chi-square example in Table 3.2-2 would lead to acceptance of the hypothesis that the data are lognormally distributed. Further testing of the hypothesis with 10 other data sets produced acceptance at the .25 probability level 9 out of 10 times and marginal acceptance at the .01 probability level the tenth time. Several tests showed probabilities greater than .75.

Both the cumulative frequency distribution and significance ratio (d-test and t-ratio) were not considered appropriate for the problem at hand. The first chi-square test, on the other hand, appeared promising but turned out to be inconclusive. However the second chi-square test indicates that the data are lognormally distributed, which is strong evidence leading toward validation of all the formulas in Section 2 (page 9).

3.2.4 Empirical

Examples 1 and 2 in Section 2.4 (page 18) are typical of the good results that were obtained by empirical testing, and similar results will be obtained by carefully using this method. That is, not only do the K factors have to be properly calculated, but care must also be exercised in adjusting the failure rates of similar equipment for differences in construction, if failure data on the same equipment is not available.

3.2.5 Data Acceptance/Rejection

Failure rates based on field experience data have been observed to vary by a factor of 10 or greater within a set of samples presumed to have come from the same population. This led to the question of whether or not to consider extreme data points as being outside the main body of data and therefore to reject them from the calculations.

This problem of inclusion or deletion of extreme data points was approached in the following manner. Techniques for processing data with extreme values when sample sizes are small were reviewed. The technique selected was an r-test (described in Chapter 16 of reference 7) which is based on a ratio comparison of the distance from the end data points to their neighbors to the total range of all the data points. This ratio establishes a probability, at a desired confidence level, that the end observation is from the same population as the others. This test also requires a normal distribution, therefore the common logarithms of the data should be used with the table and ratio formulas in Appendix IV in applying this test.

This test should be used primarily to identify data points that should be rechecked to determine, if possible, the reason for the large deviation. The decision to accept or reject an extreme data point would then be made on the basis of the recheck.

3.3 Problem Areas

Certain problems or potential problems were discovered during the course of this effort and others were pointed out by specialists in the fields of RM&SS who reviewed the method before release. Such problems are listed and discussed in this section:

3.3.1 Data Limitations
3.3.2 Application Requirements
3.3.3 Assumptions (Section 2.2)
3.3.4 Application Limitations

3.3.1 Data Limitations

Field experience data is the foundation on which this K factor development effort is built, but even though the data is the best available, it does have shortcomings.

First, the data has reporting errors. These errors can be introduced by the person reporting the failure, maintenance action, or accident/incident, or by key punch operators, or anyone else along the line of data collection. Many obvious

errors or this kind have been found and corrected to increase the validity of the data, but some remain undetected. Because most of the data is homogeneous, reporting errors will tend to cancel out in the K factor ratio process.

Sometimes there is the problem of finding field experience data for the desired application. Small data sets of 3 to 10 individual points are typical both for individual LRU's in an LRU group, used to make first level K factors, and for LRU groups in an equipment class, used to make composite K factors. However the small sample sizes are reflected in the confidence intervals associated with each K factor so that it is clear how much confidence can be placed in them.

3.3.2 Application Requirements

The greatest concern presented by RM&SS specialists who reviewed the initial draft of this K factor approach was whether or not the K factors would fit the requirements of their particular program.

The reliability people were particularly concerned with the definition of failure that would be used to determine the reliability K factors. Apparently definitions vary from program to program and even within a program. However this has no effect on the validity of this K factor development method, because the raw input data can be processed in any manner to meet the definition of failure determined by a program, and a whole new set of reliability K factors can be calculated from this data by exactly the same method. Variations in reliability K factors due to changes in failure definition have not been investigated, therefore it is possible that the definition of failure has little effect on K factors. At any rate, the method is applicable to any program.

3.3.3 Assumptions (from Section 2.2, page 10)

Several basic assumptions were outlined in Section 2.2 concerning the data used and its applicability to K factor determination and these will be further discussed in this section.

1. Experience data sets reflect an integration of all subfactors which affect reliability statistics.

2. Reliability statistics vary primarily due to environmental effects while other contributing effects tend to cancel out when K factor ratios are taken.

Because experience data reflects all factors, there will be normal random variations in failure rates due to other than the basic application effects. Since the data sets are homogeneous, these other variations will tend to cancel in the K factor ratio taking process. The added uncertainties due to these normal variations will be reflected by slightly larger confidence intervals.

3. LRU's in a general equipment class are all affected similarly by changes in application such that one composite K factor will adequately represent the entire class.

An LRU class will be affected similarly by changes in application, because equipment with similar construction will have the same modes of failure and approximately the same number of failures depending on complexity and part count. However, the equipment construction can vary considerably even within a class and a composite K factor is only an average of the entire class.

4. That a direct relationship exists between failures and operating hours (constant failure rate).

Reliability "Bath Tub" Curve

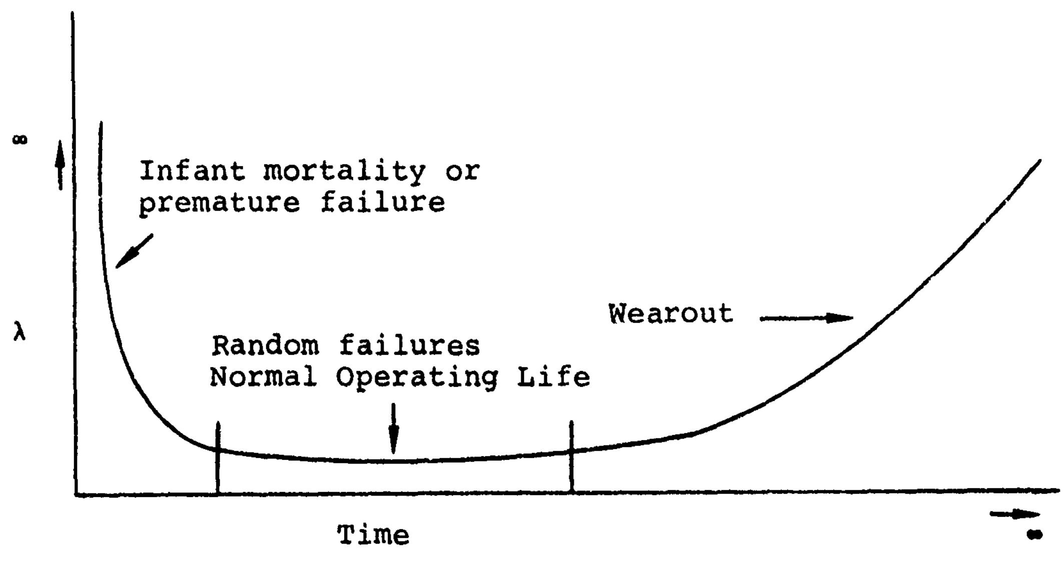

Figure 3.3-1

It is presumed that the equipment under consideration is operated in the central region of the reliability "Bath Tub" curve depicted by random failures occuring at a constant rate. This is accomplished by adequate screening and burn-in to catch the premature failures and time-scheduled removals to eliminate wearout problems. A problem comes from trying to identify the proper parameter to use as the time base. Operating hours has been used where possible but flight hours has been used for airborne environments and they both exclude storage, dormant, standby, warm up and checkout times. Further, some LRU's exhibit failures more as a function of cycles of operation than operating hours. Therefore cycles or some other measure could be better for some LRU's. However, in the past, system operating hours has proven to be a convenient base to work with and has produced satisfactory results, even though some error is introduced.

5. Failure rates are lognormally distributed.

In order to do any accurate hypothesis testing or statistical processing, it is necessary to make an assumption as to how the data is distributed. In Section 3.2.3 (page 27) an effort was made to show that failure rates appear to be lognormally distributed, which would make their common logarithms normally distributed. This is convenient because most statistical tests require that a sample be normally distributed. Because lognormality of all data sets is not proven, there may be another distribution that better describes some data sets. With little deviation from the lognormal in the samples investigated, the search for another distribution that might be better was not continued.

3.3.4 Subfactors That Impact Reliability

Application/environment K factors as developed in this study are really the integration of many parameters or subfactors which are all reflected in the field experience data, as stated in Assumption #1, Section 2.2 (page 11). It is important to recognize that these factors exist and that they do impact field failure rates, but it is not yet known how to evaluate and quantify their relative impact. Below is a list of subfactors that affect reliability. The list is not complete, but it does include many of the known subfactors.

complexity	state-of-the-art	personnel skill level
temperature	weapon system	on-off cycles
vibration	operating command	design stress level
utilization	repairability	grade of parts
duty cycle	mission type	burn-in
	phase of mission	

3.3.5 Application Limitations

K factors developed by this technique are intended for only one purpose - to predict the failure rate of a device in an environment or application for which no failure data on that device currently exists. Any use of K factors other than for the one intended would result in a trade off of accuracy for other factors, some of which could conceivably include ease in data handling or time savings.

As a general rule to follow, when a failure rate is required, available data and K factors should be used as necessary to make a prediction. Failure to use factual data in prediction has resulted in many availability, reliability, maintainability and safety problems in current systems.

4.0 CONCLUSIONS AND RECOMMENDATIONS

4.1 The following conclusions were drawn concerning the development and use of K factors:

1. A method for deriving environmental adjustment failure rate K factors has been developed.

2. The method has been validated by empirical testing against actual failure rates from field experience data.

3. The method is valid for any program or major equipment since new K factors can be calculated to fit the definitions and requirements set for each program.

4.2 Recommendations

1. In order to satisfy the requirement of processing selected data sets according to the specific definitions and requirements of each program, field experience data should be put in a mechanized file accessible by remote terminal for low cost, repetitive, rapid retrieval with convenient variable processing options.

2. Subfactors which have major impact on reliability should be investigated to determine their relative impact on the total K factor.

3. The basic technique developed in this research may be applicable to development of other types of K factors and application to other areas should be considered as the need arises and the resources become available.

NOMENCLATURE

α	- Type I error. The probability of rejecting a true hypothesis.
Application	- Intended use of an equipment (see Environment).
$\bar{\alpha}, \bar{\lambda}$	- The arithmetic mean of a sample, i.e. the sum of all observations divided by the number of observations.
β	- Type II error. The probability of accepting a false hypothesis.
Class	- A group of items alike in some way (see Equipment Classification, General).
Composite K Factor	- K factor developed from first level K factors taken from a small sampling of LRU groups within a general equipment classification and applicable to entire equipment classification.
Confidence Interval	- A range of values estimated from a random sample on the premise that the range will encompass a sought for true parameter of the sampled population a given percentage of times if the sampling process were to be repeated many times.
Confidence Level	- The percentage figure that expresses the probability or proportion of times a statement should be correct or that an estimated parameter lies within the given confidence interval.
Confidence Limits	- The upper and lower extremes of a confidence interval.
Environment	- The aggregate of all the conditions and influences which affect the operation of equipment, e.g. physical location, operating characteristics, shock, vibration, etc. Syn. application.
Equipment Classification, General	- Broadest grouping of equipment similarity, based solely on construction by predominant piece part classification. Examples: Electronic, Hydraulic, Mechanical, Electro-Mechanical, etc.

Failure Rate, λ	- A figure of merit expressing the frequency of failure occurrences which can be observed over any specified time interval or number of operating cycles; e.g. average failures per 1000 flight hours. (see MTBF)
Field [Experience] Data	- Data accumulated as a result of normal operations; as opposed to data collected from laboratory controlled tests, accelerated life tests, etc.
First Level K Factor	- A K factor developed from failure rates taken from an LRU group and applicable only to equipment within the group.
Frequency Distribution Function	- (see Probability Distribution Function).
General Equipment Classification	- (see Equipment Classification, General).
a, $\hat{\lambda}$, G. M.	- The geometric mean of a sample, i.e. the nth root of the product of n observations, (no observation can be zero).
K Factor	- 1. Any Logistics Performance factor. 2. Failure rate K factors are used to predict failure rates by utilizing failure rate data from the same/similar equipment from different applications and adjusting it for environmental differences.
Line Replaceable Unit, LRU	- 1. An equipment or assembly that is removed as a single unit and taken to a shop or similar facility for repair or maintenance. 2. A specific equipment, unique in construction and function.
LRU Group or Family	- LRU's with similar construction, similar functions and approximately equal failure rates. Failure rates from an LRU family are used to develop a single first level K factor. Examples: Hydraulic actuators, gyroscopes, check valves, etc.

MTBF	- The total number of operating hours of a population of equipments divided by the total number of failures within the population during the measured period of time. In most cases of interest, MTBF is the reciprocal of failure rate, MTBF = $1/\lambda$.
Parts, Piece Parts	- An article which is an element of an LRU or a subassembly of an LRU, and is of such construction that it is not practical or economically amenable to further disassemble for maintenance purposes. Examples: resistor, transformer, bearing.
pdf	- (see Probability Density Function)
Probability density function, pdf	- A curve or equation specifying the probability that a random variable will have a specific value.
Reliability Prediction	- To estimate beforehand the expected reliability value (failure rate) of an LRU.
Subfactors	- Identifiable effects that contribute to the overall K factor, but which have not been evaluated in this research effort. A K factor is an integration of all subfactors some of which include - utilization, duty cycle, vibration, temperature, etc.

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APPENDIX I

A computer program for use with an HP 9100A programmable calculator. Outputs are: geometric mean ($\hat{a}$), mean confidence limits ($\hat{a}_u$, $\hat{a}_L$) and/or partition boundary limits (P_u, P_L).

STEP	CODE	KEY	EXPLANATION	STEP	CODE	KEY	EXPLANATION
00	20	clear	clear	24	15	f	CONTINUE ← A
01	23	x→( )	d,e,f,x,y,z	25	27	↑	$-(\Sigma a_i)^2$
02	17	d	registers.	26	32	chg sign	
03	01	1	Set registers for start.	27	36	*	
04	27	↑	z = 1	28	12	e	
05	02	2	y = 2	29	27	↑	$n\Sigma a_i^2$
06	27	↑	x = 3	2a	17	d	
07	03	3		2b	36	*	
08	41	STOP	ENTER $x = a_i$ CONTINUE	2c	25	↓	$s_L^2 \cdot n(n-1)$
09	45	PRINT	Print a_i	2d	33	+	
0a	27	↑	Save a_i	30	17	d	
0b	17	d		31	27	↑	
0c	27	↑	Add 1 to	32	01	1	
0d	01	1	counter,	33	34	-	$n(n-1)$
10	33	+	i.	34	17	d	
11	40	y→( )		35	36	*	
12	17	d		36	25	↓	
13	22	Roll↑	Return and	37	35	÷	s_L^2
14	27	↑	repeat a_i.	38	25	↓	
15	75	log x	Accumulate	39	76	$\sqrt{x}$	
16	27	↑	log a_i in f	3a	23	x→()	STORE s_L in register b.
17	36	*	and	3b	14	b	
18	60	acc +	$(\log a_i)^2$ in e	3c	15	f	
19	30	x⟳y	Ready display. $z = a_i$, $y = \log a_i$, $x = 1$	3d	27	↑	log $\hat{a}$
1a	17	d		40	17	d	
1b	41	STOP	If entries completed	41	35	÷	
1c	43	IF FLAG	SET FLAG	42	40	y→()	STORE log $\hat{a}$ in register f.
1d	02	2	and go to next step.	43	15	f	
20	04	4					
21	44	GO TO()()	Otherwise, enter next entry.				
22	00	0					
23	11	9	$x = a_i + 1$ Continue → A				

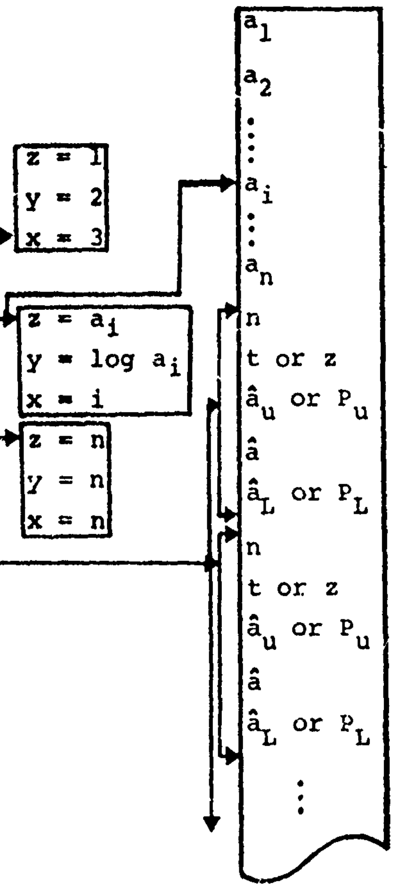
(Continued)

STEP	CODE	KEY	EXPLANATION
44	01	1	
45	00	0	
46	65	ln x	
47	36	*	$\hat{a}$
48	25	↓	
49	74	e^x	
4a	23	x→()	STORE $\hat{a}$ in
4b	12	e	register e.
4c	17	d	Ready display
4d	27	↑	z = n y = n x = n ← (B)
50	27	↑	
51	41	STOP	ENTER x=t or SET FLAG x=z CONTINUE
52	23	x→()	STORE t or z
53	16	c	in register c.
54	27	↑	Save t or z.
55	17	d	Print n.
56	45	Print	
57	16	c	Print t or z.
58	45	Print	
59	14	b	
5a	36	*	s·t or s·z
5b	43	IF FLAG	If flag is set,
5c	06	6	z test to be
5d	03	3	done.
60	17	d	
61	76	$\sqrt{x}$	$\sqrt{n}$
62	35	+	ϵ
63	27	↑	Save ϵ.
64	25	↓	

STEP	CODE	KEY	EXPLANATION
65	15	f	
66	33	÷	
67	01	1	
68	00	0	
69	65	ln x	Print $\hat{a}_u$.
6a	36	*	
6b	25	↓	
6c	74	e^x	
6d	45	Print	
70	12	e	
71	45	Print	Print $\hat{a}$.
72	15	f	
73	30	x⇄y	
74	34	−	
75	01	1	
76	00	0	
77	65	ln x	Print $\hat{a}_L$.
78	36	*	
79	25	↓	
7a	74	e^x	
7b	45	Print	
7c	44	GO TO ()()	Return for new
7d	04	4	t or z. → (B)
80	16	c	
81	46	END	End of Program..

PROGRAM EXECUTION STEPS

1. SWITCH ON.
2. PRINTER x ON.
3. SWITCH RUN.
4. GO TO 0,0.
5. SWITCH PROGRAM.
6. ENTER PROGRAM B.
7. SWITCH RUN.
8. END
9. CONTINUE
*10. ENTER $\boxed{x = a_i}$
11. CONTINUE
12. REPEAT STEPS 10→11, or
13. SET FLAG
14. CONTINUE
*15. ENTER $\boxed{x = z}$ SET FLAG, or
ENTER $\boxed{x = t}$
16. CONTINUE
17. RETURN TO STEP 15, or
18. END
19. RETURN TO STEP 9, or
20. SWITCH PRINTER OFF
21. SWITCH OFF



*Note: Use CLEAR X key only,
CLEAR key destroys program.

APPENDIX II

Condensed t-table

n-1	Two Sided Confidence Limit, % 1 - α					n-1
	80	90	95	99	99.5	
1	3.078	6.314	12.706	63.657	127.32	1
2	1.886	2.920	4.303	9.925	14.089	2
3	1.638	2.353	3.182	5.841	7.453	3
4	1.533	2.132	2.776	4.604	5.598	4
5	1.476	2.015	2.571	4.032	4.773	5
6	1.440	1.943	2.447	3.707	4.317	6
7	1.415	1.895	2.365	3.499	4.029	7
8	1.397	1.860	2.306	3.355	3.832	8
9	1.383	1.833	2.262	3.250	3.690	9
10	1.372	1.812	2.228	3.169	3.581	10
11	1.363	1.796	2.201	3.106	3.497	11
12	1.356	1.782	2.179	3.055	3.428	12
13	1.350	1.771	2.160	3.012	3.372	13
14	1.345	1.761	2.145	2.977	3.326	14
15	1.341	1.753	2.131	2.947	3.286	15
16	1.337	1.746	2.120	2.921	3.252	16
17	1.333	1.740	2.110	2.898	3.222	17
18	1.330	1.734	2.101	2.878	3.197	18
19	1.328	1.729	2.093	2.861	3.174	19
20	1.325	1.725	2.086	2.845	3.153	20
21	1.323	1.721	2.080	2.831	3.135	21
22	1.321	1.717	2.074	2.819	3.119	22
23	1.319	1.714	2.069	2.807	3.104	23
24	1.318	1.711	2.064	2.797	3.090	24
25	1.316	1.708	2.060	2.787	3.078	25
26	1.315	1.706	2.056	2.779	3.067	26
27	1.314	1.703	2.052	2.771	3.056	27
28	1.313	1.701	2.048	2.763	3.047	28
29	1.311	1.699	2.045	2.756	3.038	29
30	1.310	1.697	2.042	2.750	3.030	30
40	1.303	1.684	2.021	2.704	2.971	40
60	1.296	1.671	2.000	2.660	2.915	60
120	1.289	1.658	1.980	2.617	2.860	120
∞	1.282	1.645	1.960	2.576	2.807	∞

APPENDIX III

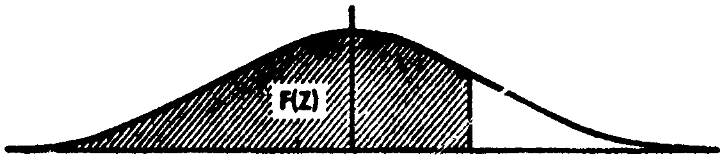


TABLE 2-II CUMULATIVE NORMAL DISTRIBUTION

z	.000	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5336	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9206	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9506	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9533
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9876	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$$F(Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz = 1 - \alpha$$

APPENDIX IV*

Statistic	Number of Observations k	Critical Values α = .30	α = .20	α = .10	α = .05	α = .02	α = .01	α = .005
$r_{10} = \frac{X_2 - X_1}{X_k - X_1}$	3	.684	.781	.886	.941	.976	.988	.994
	4	.471	.560	.679	.765	.846	.889	.926
	5	.373	.451	.557	.642	.729	.780	.821
	6	.318	.386	.482	.560	.644	.698	.740
	7	.281	.344	.434	.507	.586	.637	.680
$r_{11} = \frac{X_2 - X_1}{X_{k-1} - X_1}$	8	.318	.385	.479	.554	.631	.683	.725
	9	.288	.352	.441	.512	.587	.635	.677
	10	.265	.325	.409	.477	.551	.597	.639
$r_{21} = \frac{X_3 - X_1}{X_{k-1} - X_1}$	11	.391	.442	.517	.576	.638	.679	.713
	12	.370	.419	.490	.546	.605	.642	.675
	13	.351	.399	.467	.521	.578	.615	.649
$r_{22} = \frac{X_3 - X_1}{X_{k-2} - X_1}$	14	.370	.421	.492	.546	.602	.641	.674
	15	.353	.402	.472	.525	.579	.616	.647
	16	.338	.386	.454	.507	.559	.595	.624
	17	.325	.373	.438	.490	.542	.577	.605
	18	.314	.361	.424	.475	.527	.561	.589
	19	.304	.350	.412	.462	.514	.547	.575
	20	.295	.340	.401	.450	.502	.535	.562
	21	.287	.331	.391	.440	.491	.524	.551
	22	.280	.323	.382	.430	.481	.514	.541
	23	.274	.316	.374	.421	.472	.505	.532
	24	.268	.310	.367	.413	.464	.497	.524
	25	.262	.304	.360	.406	.457	.489	.516

* From W. J. Dixon, "Processing Data for Outliers," Biometrics, Vol. 9 (1953), p. 74.

APPENDIX V

Cumulative Chi-Square Distribution

$$F(u) = \int_0^u \frac{x^{(n-2)/2} e^{-x/2}\, dx}{2^{n/2}(n-2)/2}\,!$$

n \ F	.005	.010	.025	.050	.100	.250	.500	.750	.900	.950	.975	.990	.995
1	.0000393	.000157	.000982	.00393	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	.0717	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	.207	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	.412	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	.676	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7

APPENDIX VI
Common Logarithms

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

APPENDIX VII

Failure Rate Data for 87 Aircraft Gyroscopes

#	Flight Hours	Failures	Failures/ 1000 Hrs.	#	Flight Hours	Failures	Failures/ 1000 Hrs.
1	63,905	55	.861	45	97,318	298	3.062
2	63,905	78	1.221	46	17,787	123	6.915
3	63,905	56	.876	47	17,787	53	2.980
4	99,646	103	1.034	48	37,013	280	7.565
5	99,646	91	.913	49	37,013	129	3.485
6	99,646	117	1.174	50	52,947	352	6.648
7	156,763	74	.472	51	52,947	116	2.191
8	156,763	65	.415	52	142,190	59	.415
9	156,763	70	.447	53	142,190	18	.127
10	106,276	82	.772	54	142,190	28	.197
11	106,276	159	1.496	55	142,190	156	1.097
12	106,276	122	1.148	56	142,190	206	1.449
13	192,241	102	.531	57	142,190	138	.971
14	192,241	193	1.004	58	142,190	200	1.407
15	192,241	96	.499	59	121,609	10	.082
16	16,977	221	13.018	60	121,609	41	.337
17	33,954	15	.442	61	121,609	338	2.779
18	50,931	2	.039	62	121,609	159	1.307
19	100,319	33	.329	63	121,609	186	1.529
20	100,319	101	1.007	64	21,527	26	1.208
21	100,319	60	.598	65	43,054	68	1.579
22	20,807	80	3.845	66	43,054	11	.255
23	58,481	25	.427	67	64,581	21	.325
24	58,481	85	1.453	68	27,575	44	1.596
25	58,481	70	1.197	69	55,150	21	.381
26	20,330	88	4.329	70	55,150	63	1.142
27	79,899	60	.751	71	82,725	25	.302
28	79,899	161	2.015	72	284,382	893	3.140
29	79,899	107	1.339	73	284,382	70	.246
30	6,055	3	.495	74	568,764	29	.051
31	38,508	5	.130	75	568,764	259	.455
32	38,508	4	.104	76	853,146	1060	1.242
33	40,939	24	.586	77	70,279	294	4.183
34	40,939	49	1.197	78	70,279	11	.157
35	40,939	30	.733	79	140,558	292	2.077
36	13,950	15	1.075	80	140,558	48	.341
37	98,584	224	2.272	81	210,837	93	.441
38	98,584	37	.375	82	317,109	570	1.797
39	338,854	227	.670	83	317,109	844	2.662
40	338,854	18	.053	84	634,218	933	1.471
41	38,116	132	3.463	85	951,327	687	.722
42	38,116	24	.630	86	92,806	345	3.717
43	73,650	272	3.693	87	139,209	34	.244
44	73,650	50	.679	87	11,986,565	13,316	134.084

$\hat{x} = .855 \qquad s_L^2 = .2516$

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ACTIVE SHEET RECORD

SHEET NO	REV LTR	ADDED SHEETS SHEET NO.	REV LTR	SHEET NO.	REV LTR	SHEET NO.	REV LTR	ADDED SHEETS SHEET NO.	REV LTR	SHEET NO.	REV LTR